Model Solutions

Pearson Edexcel Level 3 GCE

Wednesday 17 June 2020

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **9FM0/3D**

Further Mathematics

Advanced

Paper 3D: Decision Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator,
Decision Mathematics Answer Book (enclosed)

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the answer book provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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P 6 2 6 7 8 A

Pearson

1. The table below shows the lengths, in km, of the roads in a network connecting seven towns, A, B, C, D, E, F and G.

	A	B	C	D	E	F	G
A	–	24	–	22	35	–	–
B	24	–	25	27	–	–	–
C	–	25	–	33	31	36	26
D	22	27	33	–	–	42	–
E	35	–	31	–	–	37	29
F	–	–	36	42	37	–	40
G	–	–	26	–	29	40	–

(a) By adding the arcs from vertex D along with their weights, complete the drawing of the network on Diagram 1 in the answer book.

(2)

(b) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order that you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.

(3)

(c) State the weight of the minimum spanning tree.

(1)

(Total for Question 1 is 6 marks)

1.

	A	B	C	D	E	F	G
A	–	24	–	22	35	–	–
B	24	–	25	27	–	–	–
C	–	25	–	33	31	36	26
D	22	27	33	–	–	42	–
E	35	–	31	–	–	37	29
F	–	–	36	42	37	–	40
G	–	–	26	–	29	40	–

a)

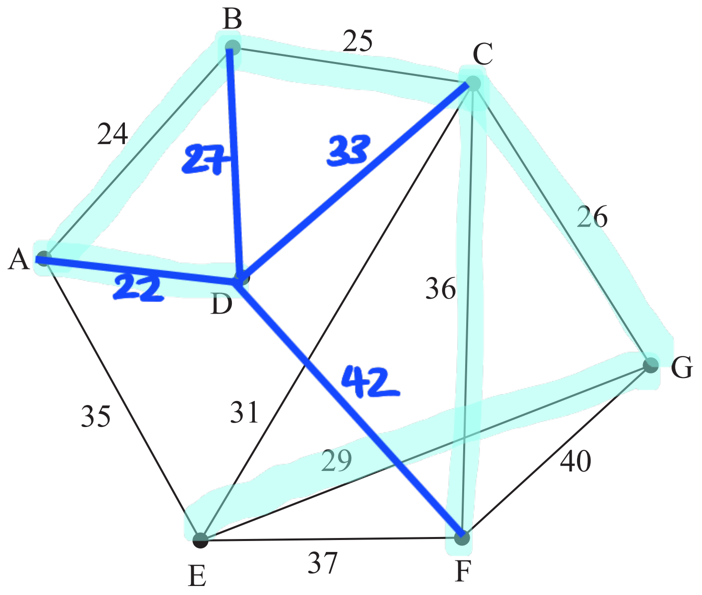


Diagram 1

b) AD – 22 ✓
AB – 24 ✓
BC – 25 ✓
CG – 26 ✓
BD – 27 ✗ reject (makes a cycle)
EG – 29 ✓
CE – 31 ✗ reject "
CD – 33 ✗ reject "
AE – 35 ✗ reject "
CF – 36 ✓
EF – 37 ✗ reject "
FG – 40 ✗ reject "
DF – 42 ✗ reject. "

c) weight = 22+24+25+26+29+36
= 162 km

P 6 2 6 7 8 A 0 2 2 0

2.

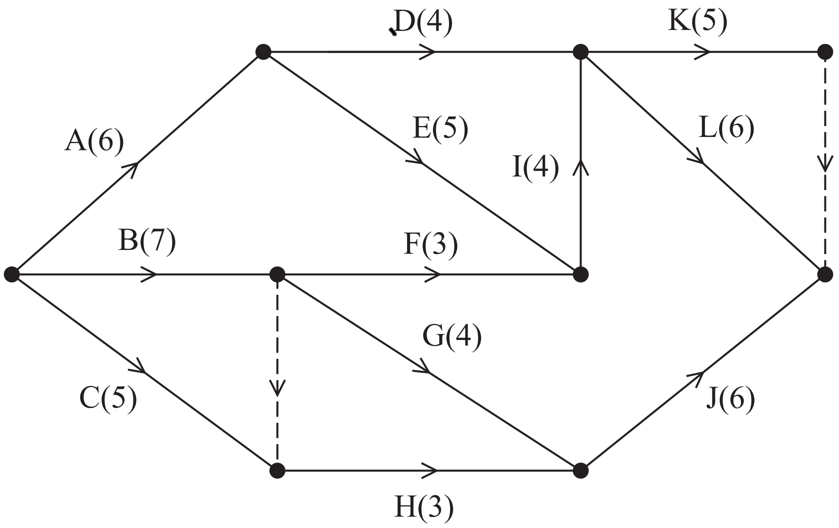


Figure 1

The network in Figure 1 shows the activities that need to be undertaken to complete a project. Each activity is represented by an arc and the duration, in hours, of the corresponding activity is shown in brackets.

(a) Explain why each of the dummy activities is required.

(2)

(b) Complete the table in the answer book to show the immediately preceding activities for each activity.

(2)

(c) (i) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(ii) State the minimum completion time for the project.

(iii) State the critical activities.

(6)

Each activity requires one worker. Each worker is able to do any of the activities. Once an activity is started it must be completed without interruption.

(d) On Grid 1 in the answer book, draw a resource histogram to show the number of workers required at each time when each activity begins at its earliest possible start time.

(3)

(e) Determine whether or not the project can be completed in the minimum possible time using fewer workers than the number indicated by the resource histogram in (d). You must justify your answer with reference to the resource histogram and the completed Diagram 1.

(2)

(Total for Question 2 is 15 marks)

2. (a)

The dummy at the end of activity B is required as F and G are dependent on only activity B, but activity H is dependent on activities B and C.

The dummy at the end of activity K is required as 2 activities cannot start at the same event and finish at the same event.

(b)

Activity	Immediately preceding activities	Activity	Immediately preceding activities	Activity	Immediately preceding activities
A	–	E	A	I	E, F
B	–	F	B	J	G, H
C	–	G	B	K	D, I
D	A	H	B, C	L	D, I

ci)

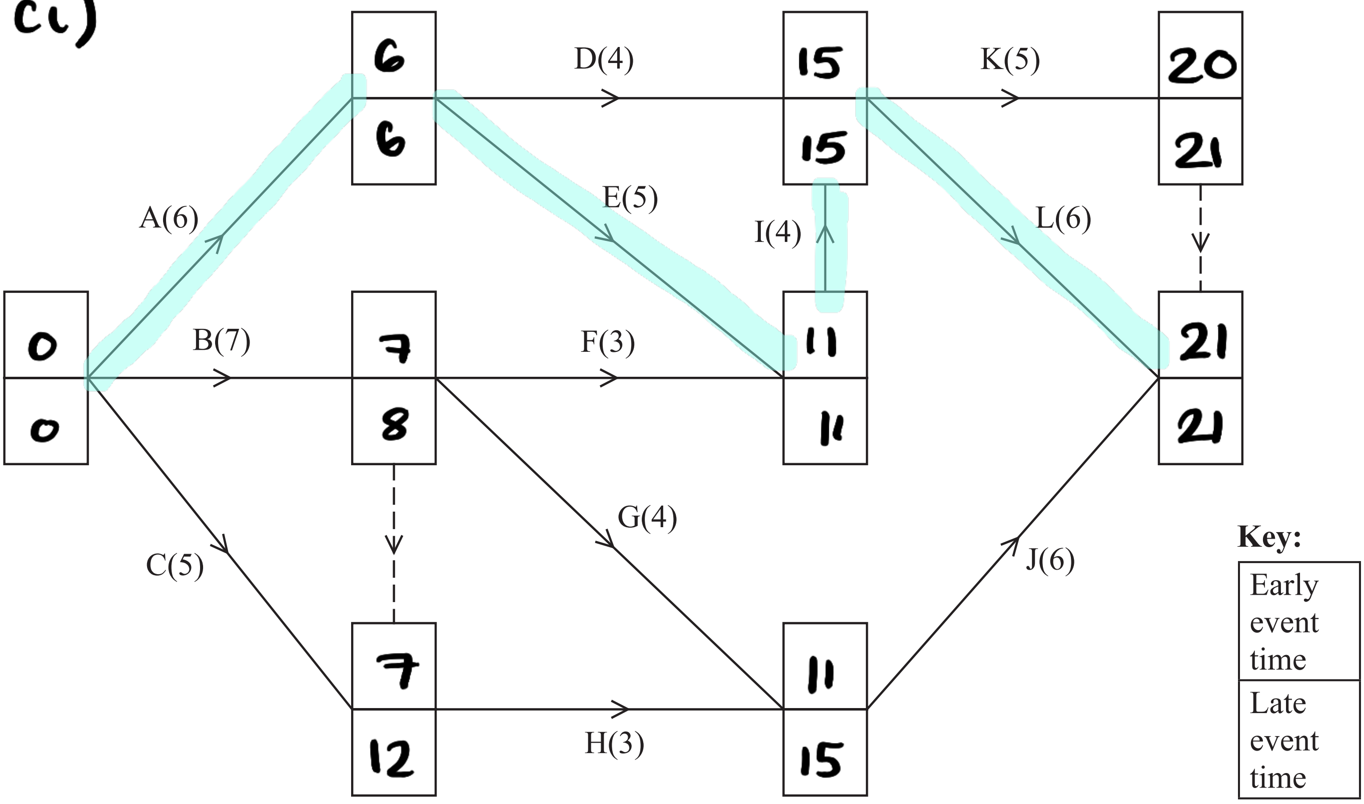
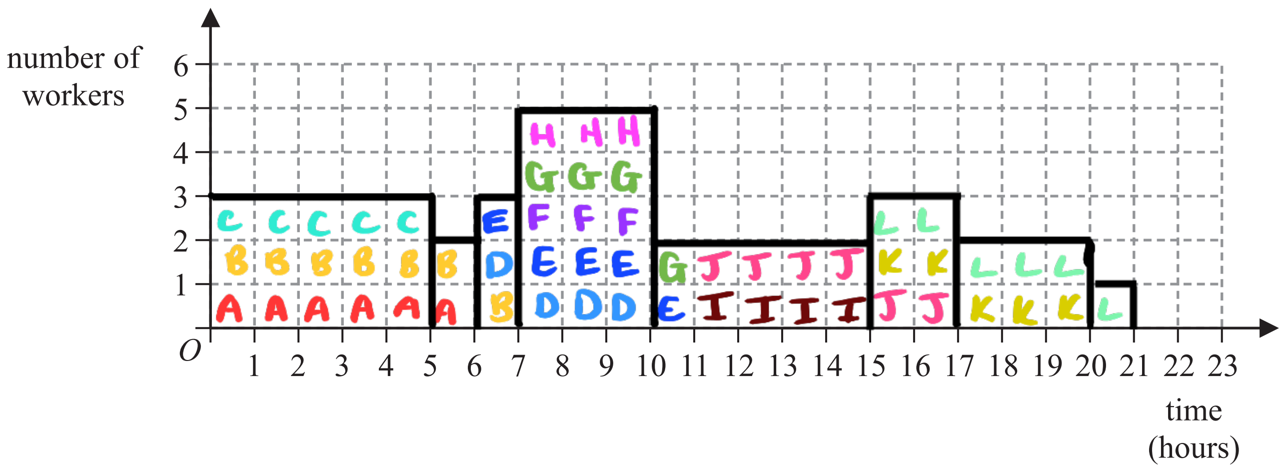


Diagram 1

Question 2 continued

c ii) min. completion time : 21 hours

c iii) Critical activities: A, E, I, L



Grid 1

e) Between 7 and 10 (hours), there are currently 5 workers. To reduce the amount of workers we can move one of the non-critical activities D, F, G or H until after time 10.

(Total for Question 2 is 15 marks)

3.

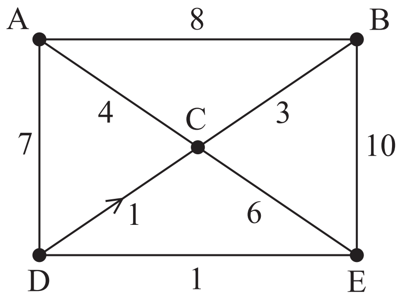


Figure 2

Direct roads between five villages, A, B, C, D and E, are shown in Figure 2. The weight on each arc is the time, in minutes, it takes to travel along the corresponding road. The road from D to C is one-way as indicated by the arrow on the corresponding arc.

Floyd's algorithm is to be used to find the complete network of shortest times between the five villages.

(a) Set up initial time and route matrices.

(2)

The matrices after two iterations of Floyd's algorithm are shown below.

Time matrix

	A	**B**	**C**	**D**	**E**
A	–	8	4	7	18
B	8	–	3	15	10
C	4	3	–	11	6
D	7	15	1	–	1
E	18	10	6	1	–

Route matrix

	A	**B**	**C**	**D**	**E**
A	A	B	C	D	B
B	A	B	C	A	E
C	A	B	C	A	E
D	A	A	C	D	E
E	B	B	C	D	E

(b) Perform the next two iterations of Floyd's algorithm that follow from the tables above. You should show the time and route matrices after each iteration.

(4)

The final time matrix after completion of Floyd's algorithm is shown below.

Final time matrix

	A	**B**	**C**	**D**	**E**
A	–	7	4	7	8
B	7	–	3	10	9
C	4	3	–	7	6
D	5	4	1	–	1
E	6	5	2	1	–

3. (a)

Initial time matrix

	A	B	C	D	E
A	–	8	4	7	∞
B	8	–	3	∞	10
C	4	3	–	∞	6
D	7	∞	1	–	1
E	∞	10	6	1	–

Initial route matrix

	A	B	C	D	E
A	A	B	C	D	E
B	A	B	C	D	E
C	A	B	C	D	E
D	A	B	C	D	E
E	A	B	C	D	E

(b)

Time matrix

	A	B	C	D	E
A	–	[7]	4	7	[10]
B	[7]	–	3	[14]	[9]
C	4	3	–	11	6
D	[5]	[4]	1	–	1
E	[10]	[9]	6	1	–

Route matrix

	A	B	C	D	E
A	A	C	C	D	C
B	C	B	C	C	C
C	A	B	C	A	E
D	C	C	C	D	E
E	C	C	C	D	E

Time matrix

	A	B	C	D	E
A	–	7	4	7	[8]
B	7	–	3	14	9
C	4	3	–	11	6
D	5	4	1	–	1
E	[6]	[5]	[2]	1	–

Route matrix

	A	B	C	D	E
A	A	C	C	D	D
B	C	B	C	C	C
C	A	B	C	A	E
D	C	C	C	D	E
E	D	D	D	D	E

If you make an error there are spare copies of these matrices on Page 9.

(c) (i) Use the nearest neighbour algorithm, starting at A, to find a Hamiltonian cycle in the complete network of shortest times.

(ii) Find the time taken for this cycle.

(iii) Interpret the cycle in terms of the actual villages visited.

(3)

(Total for Question 3 is 9 marks)

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ci) $A \overset{4}{} C \overset{3}{} B \overset{9}{} E \overset{1}{} D \overset{5}{} A$

cii) $4 + 3 + 9 + 1 + 5 = 22$ minutes

ciii) A – C – B – C – E – D – C – A

4.

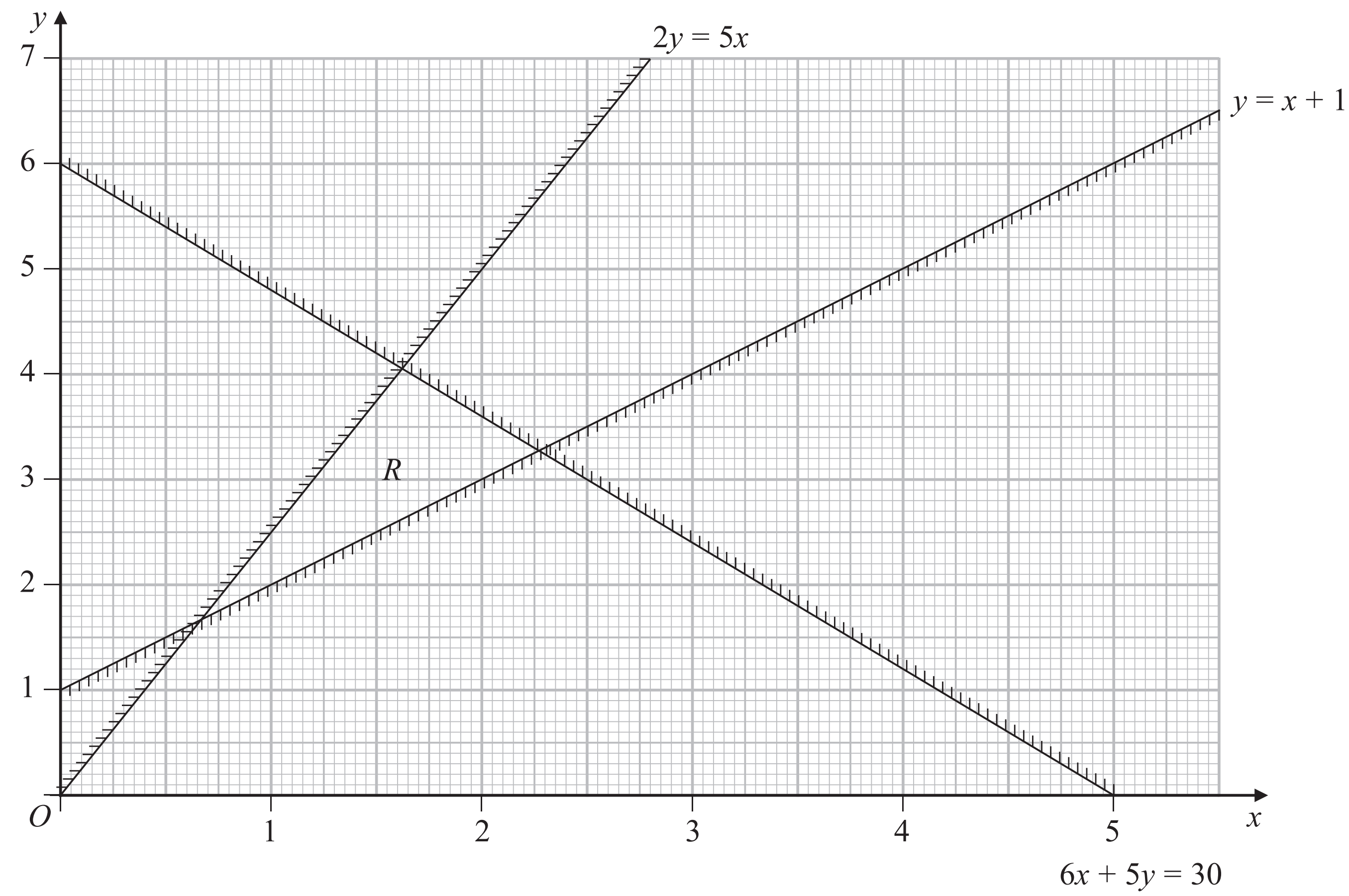

Figure 3

Figure 3 shows the constraints of a linear programming problem in x and y, where R is the feasible region.

(a) Write down the inequalities that define R.

(2)

The objective is to maximise P, where $P = 3x + y$

(b) Obtain the exact value of P at each of the three vertices of R and hence find the optimal vertex, V.

(4)

The objective is changed to maximise Q, where $Q = 3x + ay$. Given that a is a constant and the optimal vertex is still V,

(c) find the range of possible values of a.

(4)

(Total for Question 4 is 10 marks)

4.

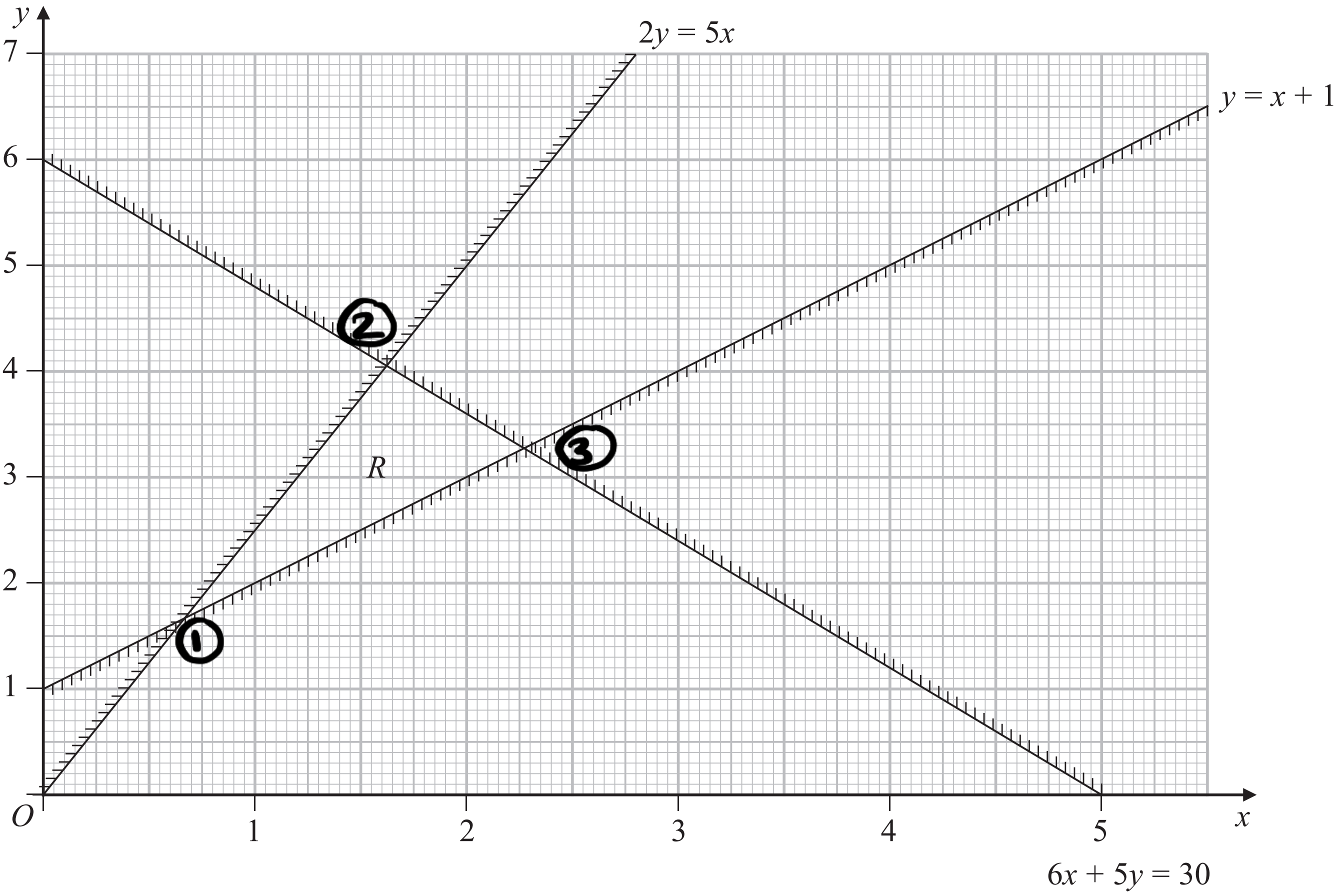

Figure 3

a) $2y \leq 5x$

$y \geq x+1$

$6x + 5y \leq 30$

b) $2y = 5x$

$2(x+1) = 5x$

$3x = 2$

$\frac{2}{3} = x$, $y = \frac{5}{3}$

① $\left(\frac{2}{3}, \frac{5}{3}\right)$

$2y = 5x$

$6x + 5y = 30$

$x = \frac{60}{37}$, $y = \frac{150}{37}$

② $\left(\frac{60}{37}, \frac{150}{37}\right)$

$y = x+1$

$6x + 5y = 30$

$x = \frac{25}{11}$, $y = \frac{36}{11}$

③ $\left(\frac{25}{11}, \frac{36}{11}\right)$

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P 6 2 6 7 8 A 0 1 0 2 0

Question 4 continued

when $x = \frac{2}{3}$, $y = \frac{5}{3}$

$P = 3\left(\frac{2}{3}\right) + \frac{5}{3} = \frac{11}{3}$

when $x = \frac{60}{37}$, $y = \frac{150}{37}$

$P = 3\left(\frac{60}{37}\right) + \frac{150}{37} = \frac{330}{37}$

when $x = \frac{25}{11}$, $y = \frac{36}{11}$

$P = 3\left(\frac{25}{11}\right) + \frac{36}{11} = \frac{111}{11}$ (largest value)

$\therefore$ Optimal vertex is $\left(\frac{25}{11}, \frac{36}{11}\right)$

c) $Q = 3x + ay$

$3\left(\frac{25}{11}\right) + a\left(\frac{36}{11}\right) > 3\left(\frac{60}{37}\right) + a\left(\frac{150}{37}\right)$

$\frac{75}{11} + \frac{36a}{11} > \frac{180}{37} + \frac{150a}{37}$

$\Rightarrow a < \frac{5}{2}$

$3\left(\frac{25}{11}\right) + a\left(\frac{36}{11}\right) > 3\left(\frac{2}{3}\right) + a\left(\frac{5}{3}\right)$

$\frac{75}{11} + \frac{36a}{11} > 2 + \frac{5a}{3}$

$\Rightarrow a > -3$

5. The nine distinct numbers in the following list are to be packed into bins of size 50

23 17 19 x 24 8 18 10 21

When the first-fit bin packing algorithm is applied to the numbers in the list it results in the following allocation.

Bin 1: 23 17 8

Bin 2: 19 x 10

Bin 3: 24 18

Bin 4: 21

(a) Explain why $13 < x < 21$

(3)

The same list of numbers is to be sorted into descending order. A bubble sort, starting at the left-hand end of the list, is to be used to obtain the sorted list. After the first complete pass the list is

23 19 17 24 x 18 10 21 8

(b) Using this information, write down the smallest interval that must contain x, giving your answer as an inequality.

(2)

When the first-fit decreasing bin packing algorithm is applied to the nine distinct numbers it results in the following allocation.

Bin 1: 24 23

Bin 2: 21 19 10

Bin 3: 18 17 x

Bin 4: 8

Given that only one of the bins is full and that x is an integer,

(c) calculate the value of x. You must give reasons for your answer.

(2)

(Total for Question 5 is 7 marks)

5. 23 17 19 x 24 8 18 10 21

a) If x has been placed in Bin 2 then $10 < x \leq 31$
As, Bin 1 at this stage only contains 40 and before x has been placed in Bin 2 it only contained 19.

As the '18' has been placed in Bin 3 this implies that $x > 50-(19+18)$ so $x > 13$.
As the '10' has been placed in Bin 2 after the x then $x \leq 50-(19+10)$ so $x \leq 21$.

However, the number are all distinct $\therefore$ $13 < x < 21$.

b) $13 < x < 17$

c) As x is placed in Bin 3, then $x \leq 50-(17+18)$
$\Rightarrow x \leq 15$.

Only 1 bin is full, which is Bin 2 $(21+19+10=50)$.
$\therefore$ x cannot be equal to 15.
x cannot be anything less or equal to 13,
$\therefore$ $x = 14$.

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P 6 2 6 7 8 A 0 1 4 2 0

6.

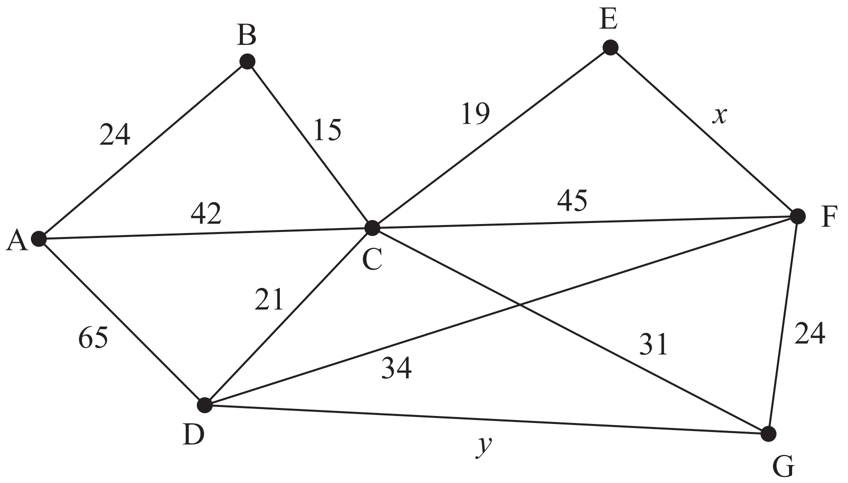


Figure 4

[*The total weight of the network is 320 + x + y*]

(a) State, with justification, whether the graph in Figure 4 is Eulerian, semi-Eulerian or neither.
(2)

The weights on the arcs in Figure 4 represent distances. The weight on arc EF is x where $12 < x < 26$ and the weight on arc DG is y where $0 < y < 10$

An inspection route of minimum length that traverses each arc at least once is found. The inspection route starts and finishes at A and has a length of 409

It is also given that the length of the shortest route from F to G via A is 140

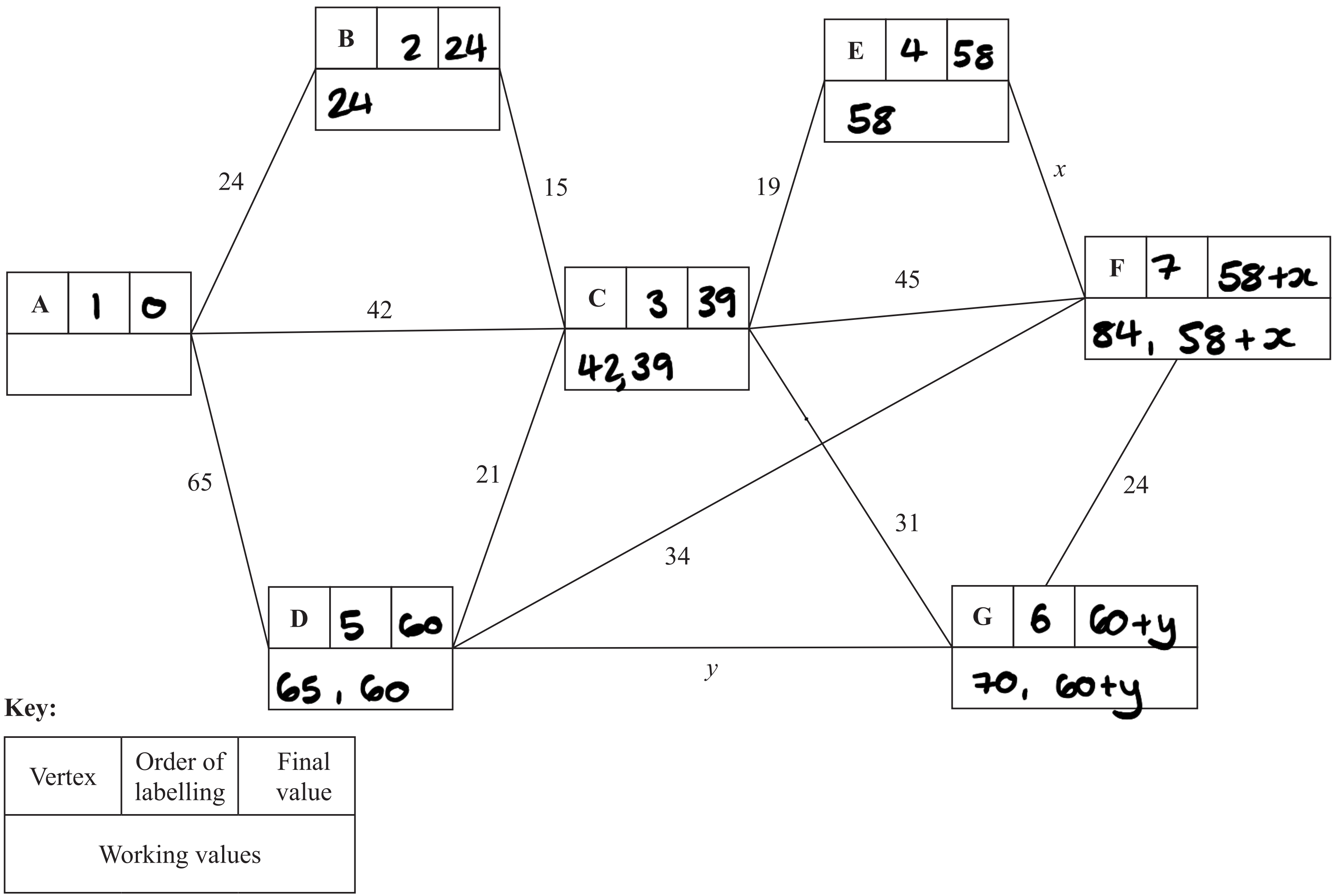
(b) Using appropriate algorithms, find the value of x and the value of y.
(9)

(Total for Question 6 is 11 marks)

6. (a)

Graph contains exactly two odd nodes ∴ graph is semi Eulerian.

b)



P 6 2 6 7 8 A 0 1 6 2 0

b continued)

Shortest path from A to F: $58+x$
Shortest path from A to G: $60+y$

$58+x+60+y=140$
$x+y=22$ — (1)

Odd nodes: A, G

$AG = 60+y$
AG is transversed twice

∴ Weight of new route:
$(320+x+y)+(60+y)=409$
$x+2y=29$ — (2)

(1) − (2):
$x+y-(x+2y)=22-29$
$-y=-7$

$y=7$, $x=15$

7. A maximisation linear programming problem in x, y and z is to be solved using the two-stage simplex method.

The partially completed initial tableau is shown below.

Basic variable	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	1	2	3	1	0	0	0	0	45
a_1	3	2	0	0	–1	0	1	0	9
a_2	–1	0	4	0	0	–1	0	1	4
P	–2	–1	–3	0	0	0	0	0	0
A									

(a) Using the information in the above tableau, formulate the linear programming problem. State the objective and list the constraints as inequalities.

(4)

(b) Complete the bottom row of Table 1 in the answer book. You should make your method and working clear.

(2)

The following tableau is obtained after two iterations of the first stage of the two-stage simplex method.

Basic variable	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	0	$\frac{5}{6}$	0	1	$\frac{7}{12}$	$\frac{3}{4}$	$-\frac{7}{12}$	$-\frac{3}{4}$	$\frac{147}{4}$
x	1	$\frac{2}{3}$	0	0	$-\frac{1}{3}$	0	$\frac{1}{3}$	0	3
z	0	$\frac{1}{6}$	1	0	$-\frac{1}{12}$	$-\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{7}{4}$
P	0	$\frac{5}{6}$	0	0	$-\frac{11}{12}$	$-\frac{3}{4}$	$\frac{11}{12}$	$\frac{3}{4}$	$\frac{45}{4}$
A	0	0	0	0	0	0	1	1	0

(c) (i) Explain how the above tableau shows that a basic feasible solution has been found for the original linear programming problem.

(ii) Write down the basic feasible solution for the second stage.

(3)

(d) Taking the most negative number in the profit row to indicate the pivot column, perform one complete iteration of the second stage of the two-stage simplex method, to obtain a new tableau, T. Make your method clear by stating the row operations you use.

(5)

(e) (i) Explain, using T, whether or not an optimal solution to the original linear programming problem has been found.

(ii) Write down the value of the objective function.

(iii) State the values of the basic variables.

(3)

(Total for Question 7 is 17 marks)

TOTAL FOR PAPER IS 75 MARKS

7.

a) $P_{max} = 2x + y + 3z$

- $x + 2y + 3z \leq 45$
- $3x + 2y \geq 9$
- $-x + 4z \geq 4$

b) $x + 2y + 3z + s_1 = 45$

$3x + 2y - s_2 + a_1 = 9 \Rightarrow a_1 = 9 - 3x - 2y + s_2$

$-x + 4z - s_3 + a_2 = 4 \Rightarrow a_2 = 4 + x - 4z + s_3$

$A = -(a_1 + a_2) = -(9 - 3x - 2y + s_2 + 4 + x - 4z + s_3)$

$= -(13 - 2x - 2y - 4z + s_2 + s_3)$

$\therefore A - 2x - 2y - 4z + s_2 + s_3 = -13$

Basic variable	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	1	2	3	1	0	0	0	0	45
a_1	3	2	0	0	–1	0	1	0	9
a_2	–1	0	4	0	0	–1	0	1	4
P	–2	–1	–3	0	0	0	0	0	0
A	-2	-2	-4	0	1	1	0	0	-13

Table 1

ci) In the tableau, the value of the objective A equals to zero, implying that the basic feasible solution has been found.

cii) $x = 3$, $y = 0$, $z = \frac{7}{4}$, $s_1 = \frac{147}{4}$, $s_2 = s_3 = 0$

Question 7 continued

b.v.	x	y	z	s_1	s_2	s_3	Value
s_1	0	$\frac{5}{6}$	0	1	$\frac{7}{12}$	$\frac{3}{4}$	$\frac{147}{4}$
x	1	$\frac{2}{3}$	0	0	$-\frac{1}{3}$	0	3
z	0	$\frac{1}{6}$	1	0	$-\frac{1}{12}$	$-\frac{1}{4}$	$\frac{7}{4}$
P	0	$\frac{5}{6}$	0	0	$-\frac{11}{12}$	$-\frac{3}{4}$	$\frac{45}{4}$

b.v.	x	y	z	s_1	s_2	s_3	Value	Row Ops
s_2	0	10/7	0	12/7	1	9/7	63	$R1 \div \frac{7}{12}$
x	1	8/7	0	4/7	0	3/7	24	$R2 + \frac{1}{3}R1$
z	0	2/7	1	1/7	0	-1/7	7	$R3 + \frac{1}{12}R1$
P	0	15/7	0	11/7	0	3/7	69	$R4 + \frac{11}{12}R1$

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b.v.	x	y	z	s_1	s_2	s_3	Value	Row Ops
P								

ei) An optimal solution has been found as there is no negative values in the objective P row.

eii) P = 69

P 6 2 6 7 8 A 0 1 9 2 0

Turn over

Question 7 continued

e iii) $S_2 = 63$, $x = 24$, $z = 7$.

(Total for Question 7 is 17 marks)

TOTAL FOR PAPER IS 75 MARKS

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